Visual Control of Braking: A Test of the $\tau$ Hypothesis

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Deceleration during braking could be controlled by (a) using the time derivative of the relative rate of optical expansion, relative to a $-0.5$ margin value of tau-dot (D. N. Lee, 1976) or (b) computing the required deceleration from spatial variables (i.e., perceived distance, velocity, or object size). Participants viewed closed-loop displays of approach to an object and regulated their deceleration with a brake. The object appeared on a checkerboard ground surface (providing velocity, distance, and size information) or with no background (providing only optical expansion). Mean tau-dot during braking was $-0.51$, and estimates of the critical value of tau-dot based on brake adjustments were $-0.44$ and $-0.52$, close to the expected value. There were no overall effects of the ground surface or object size. The results are consistent with a tau-dot strategy, where the direction and magnitude of brake adjustments are regulated using tau-dot.

How might one visually regulate braking to stop in front of an object? This is a general problem of locomotor control that arises in many situations in human experience, such as stopping behind another vehicle during driving and slowing down to open a door or greet another person during walking. It is also relevant in nonhuman situations, such as decelerating to land in insect and bird flight and collision avoidance in mobile robots and intelligent vehicles. Lee (1976) originally proposed that the time derivative of the relative rate of optical expansion ($\tau$), a second-order variable, could be used to control deceleration during braking. The experiment reported here tests this and several competing hypotheses about the visual control of braking, using an active closed-loop task.

Movement of an observer with respect to other surfaces produces patterns of optic flow that may reciprocally be used to control locomotion (Gibson, 1979; Gibson, Olum, & Rosenblatt, 1955), including information about the 3-D structure of the environment (Rogers & Graham, 1979) and the observer’s self-motion (Warren, Morris, & Kalish, 1988). There are a number of optical properties that could be used in the control of braking. One such property is optical expansion: As an observer approaches an object, its visual angle increases geometrically (see Figure 1). Lee (1976, 1980) proved that during a direct approach at constant velocity, the time-to-contact with the object ($T_c$) is specified by the inverse of its relative rate of expansion ($\tau$)

$$\tau = \frac{\theta}{\dot{\theta}} = T_c,$$

where $\theta$ is the visual angle between any two points on the object and $\dot{\theta}$ is the rate of change in that visual angle (see Tresilian, 1990, 1991, for limitations on this analysis). There is considerable evidence that humans and other animals rely on some version of the tau variable to judge time-to-contact and initiate appropriate actions (Bootsma & van Weeringen, 1990; Kaiser & Mowafy, 1993; Lee & Reddish, 1981; Lee, Young, Reddish, Lough, & Clayton, 1983; Savelsbergh, Whiting, & Bootsma, 1991; Schiff & Oldak, 1990; Todd, 1981).

Lee (1976) further showed that the time derivative of $\tau$ ($\dot{\tau}$, or “tau-dot”) could be used to control deceleration during braking. Tau-dot is a dimensionless variable that specifies the rate of change in time-to-contact and provides information about whether one’s current level of deceleration is sufficient to stop in front of an object. For example, a tau-dot value of $-0.3$ indicates a current deceleration that is slightly too high, such that, if it were maintained, the observer would stop short of the object. Hence, the observer is not in a “safe” state and could reduce deceleration. Conversely, a value of $-0.6$ indicates deceleration that is too low and, if maintained, would result in a collision. Thus, the observer is in a “crash” state and should increase deceleration. Finally, a value of $-0.5$ indicates that the current level of deceleration will bring the observer to a stop right at the object.

The present experiment tested the tau-dot hypothesis...
against several competing hypotheses, including the use of optical expansion ($\theta$) per se and the use of information about the 3-D environmental structure such as distance, size, and velocity. We describe four classes of strategies, discuss the existing evidence, and report our experimental test.

Possible Strategies

**Strategy 1: Tau-Dot ($\dot{\tau}$)**

There are several ways in which tau-dot could be used as a control variable to regulate braking. We describe four possible strategies, two of which are implicit in Lee's (1976) work.

*a. Hold tau-dot constant at $\dot{\tau}_m \approx -0.5$.** Lee (1976, p. 446) originally proposed that the observer could “attempt to maintain $\dot{\tau}$ at a safe margin value $\dot{\tau}_m$. To illustrate the consequences of this strategy for the observer's motion, we performed numerical simulations showing the effect of holding tau-dot constant at various values, assuming initial conditions in the middle of the range of the present study ($v_0 = 12.4$ m/s [30 mph], $z_0 = 49.5$ m, $\tau_0 = 4$ s; see Appendix). Sample velocity and deceleration profiles are presented in Figure 2.

First, note that $\dot{\tau} = -1.0$ corresponds to a constant velocity with no deceleration, resulting in a crash. Second, holding tau-dot constant at any value greater than $-1.0$ would, in the limit, bring the participant to a smooth stop right at the object. However, this is impractical for most values of tau-dot because they would either require an infinite deceleration or result in long, protracted approaches. With $-1.0 < \dot{\tau} < -0.5$, the approach time is short, but an infinite deceleration is required at the end of the approach (Figure 2). To stop successfully, a second phase of braking would be required, arbitrarily increasing deceleration earlier in the approach. For instance, a constant $\dot{\tau} = -0.6$ would yield a stop in under 7 s, and required deceleration would remain moderate (under 0.3 g) up until the last half-second, but would then explode to infinity. On the other hand, with $-0.5 < \dot{\tau} < 0$, an opposite problem arises: Decelerations are moderate, but as tau-dot gets larger the duration of the approach increases dramatically, becoming asymptotic at $\dot{\tau} = 0$. For instance, a constant $\dot{\tau} = -0.3$ would require an initial deceleration of only 0.22 g under our conditions, but would take about 14 s to stop. For all $\dot{\tau} \geq 0$, approach time is infinite.

A value of $\dot{\tau}_m = -0.5$ is a special case that produces a constant deceleration to a stop at the object, with the precise value of deceleration depending on the point in the approach at which braking is initiated. It is thus the smoothest and most efficient braking strategy, producing a comfortable stop in a reasonable time with no “jerk” or sudden changes in force. Under our conditions, it would yield a constant deceleration of about 0.16 g and an approach time of 8 s. Of course, a cautious observer who wanted to approach slowly could use a margin value of $-0.3$, whereas a risk-taker seeking a faster approach might adopt a margin value of $-0.6$. It is clear from Figure 2 that a reasonable braking strategy would be to use a margin value somewhere between $-0.4$ and $-0.5$ in order to avoid both infinite decelerations and a protracted approach.

In sum, $\dot{\tau} < -0.5$ would require an infinite final deceleration and result in a collision, whereas $\dot{\tau} > -0.5$ guarantees a gradual approach to a stop. Holding tau-dot constant at $\dot{\tau}_m = -0.5$ would result in a constant deceleration and a smooth, timely stop. Obviously, given the dynamics of a control loop, it is not possible for an observer to hold tau-dot strictly constant. Strategy 1a would then essentially treat $\dot{\tau}_m$ as a set point in a feedback loop, and deceleration would be continuously adjusted to maintain tau-dot as close to $-0.5$ as possible. In this case, both the onset and offset of brake adjustments should be contingent on the current value of tau-dot relative to the margin value. Alternatively, brake adjustments could be ballisitic, as in the next two strategies.

*b. Control direction of adjustment from $\dot{\tau}_m \approx -0.5$.** In practice, the observer need not maintain a fixed value of tau-dot as long as the required deceleration remains within bounds. Given that tau-dot provides information about the adequacy of current deceleration, observers could use it in a qualitative manner to determine the direction of a discrete adjustment. A $\dot{\tau} > -0.5$ indicates that deceleration is too high and should be reduced (possibly by a fixed amount), whereas $\dot{\tau} < -0.5$ indicates that deceleration is too low and should be increased (see Figure 2 in Lee, 1976). The strategy can be expressed as follows:

\[
\text{if } \dot{\tau} < -0.5, +\Delta d,
\]
\[
\text{if } \dot{\tau} > -0.5, -\Delta d.
\]

This strategy would yield closed-loop behavior with a mean $\tau$ near $-0.5$ and brake adjustments that tend to move $\dot{\tau}$ in the direction of $-0.5$, although the offset of adjustment would not occur when tau-dot reached its margin value. Individual trials could diverge widely from the ideal deceleration profile.
Figure 2. Simulation of deceleration and velocity profiles for constant values of \( \tau \). Initial conditions: \( z_0 = 49.5 \) m, \( \tau_0 = 4.0 \) s, \( v_0 = -12.4 \) m/s.

c. Control direction and magnitude of adjustment from \( \tau_m \approx -0.5 \). The current value of tau-dot provides information about not only the required direction of adjustment but also the required magnitude of adjustment. The difference between the current value of tau-dot and the margin value of \( -0.5 \) gives the required change in tau-dot that will yield the optimal braking pattern:

\[ \Delta \tau = (\tau_m - \tau). \]  

This specifies both the direction and magnitude of a discrete brake adjustment for any current value of tau-dot, as follows. The relation between tau-dot and deceleration is \( d = (v/\tau)(\tau + 1) \), based on Equation A5 (see Appendix). During braking, \( v \) and \( \tau \) tend to decrease together, so deceleration is approximately proportional to tau-dot for values around \( -0.5 \), except near the end of the approach (top panel of Figure 2). Tau-dot can thus be used to control the required change in deceleration:

\[ \Delta d \propto (\tau_m - \tau). \]  

This strategy would yield closed-loop behavior with a mean \( \tau \) near \( -0.5 \) and adjustments that are proportional to the current distance from \( \tau_m = -0.5 \). Individual trials may diverge significantly from the ideal deceleration profile, but would tend toward successive approximations of \( -0.5 \).

d. Control direction of adjustment from the direction of drift in \( \tau \). A fourth possible strategy is similar to Strategy 1b in its qualitative character but uses what is technically a different source of information. When deceleration is held constant at any value other than \( -0.5 \), \( \tau \) drifts away from \( -0.5 \). If the current \( \tau \) is greater than \( -0.5 \) and deceleration is constant, \( \tau \) will drift upward with a positive slope, such that \( \tau > 0 \). On the other hand, if \( \tau \) is currently below \( -0.5 \) and deceleration is held constant, \( \tau \) will drift downward with a negative slope, such that \( \tau < 0 \). The observer could thus hold the brake at a constant position, determine the direction of \( \tau \) drift (the sign of \( \tau \)) rather than the value of \( \tau \) per se, and use this to determine the direction in which to move the brake:

\[ \begin{align*}
\text{if } \tau < 0, & \quad +\Delta d, \\
\text{if } \tau > 0, & \quad -\Delta d.
\end{align*} \]  

This strategy implies that the visual system can reliably extract a third-order variable with some precision. It requires that a detectable change in tau-dot occur before the appropriate adjustment can be made, presumably obeying Weber’s law (\( \Delta \tau/\tau = c \)).

Strategy 2: Rate of Expansion (\( \theta \))

Another strategy that would lead to a successful stop is to hold the rate of expansion per se at any constant positive value \( \theta = k \). Simulated velocity and deceleration profiles are shown in Figure 3, for our experimental conditions (the derivation is given in the Appendix). The observer would adjust deceleration to keep the simple rate of expansion of the visual angle of the object at a constant value \( k \) (or equivalently, its second derivative could be held equal to zero, \( \theta = 0 \)). This would automatically result in a smooth stop at the object. The value of \( k \) depends on the point in the approach at which braking is initiated. However, this strategy has two drawbacks. First, the initial decelerations required are extremely large; to reduce them, the observer must initiate braking quite early in the approach. Second, the approach is nearly asymptotic; approach time increases as braking is initiated earlier, to keep deceleration within bounds. On the face of it, this does not appear to be a promising strategy.

We can predict the sorts of \( \tau \) values that would be expected under the \( \theta \) strategy (see Appendix), allowing us to test it empirically. For most of the approach, \( \tau \) would be
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Figure 3. Simulation of deceleration and velocity profiles for constant values of $\theta$. Initial conditions: $z_0 = 49.5$ m, $\tau_0 = 4.0$ s, $v_0 = -12.4$ m/s, radius = 0.66 m. Each curve represents a different constant expansion rate $k$, corresponding to onset of braking at 0%, 25%, 50%, or 75% of the way from the initial distance.

The following strategies appear to be the simplest, although others can also be derived.

$\text{a. Compute deceleration from perceived distance (z) and velocity (v):}$

$$d = \frac{v^2}{2z}.$$  

$\text{b. Compute deceleration from perceived distance (z) and } \tau:$

$$d = z/\tau^2.$$  

$\text{c. Compute deceleration from perceived velocity (v) and } \tau:$

$$d = \frac{v}{\tau}.$$  

$\text{d. Compute deceleration from perceived object size (r), visual angle (\theta), and } \tau:$

$$d = \frac{r}{\tau^2 \tan \theta}.$$  

$\text{e. Compute deceleration from perceived object size (r), visual angle (\theta) and velocity (v):}$

$$d = \frac{\sqrt{v^2 \tan \theta}}{2r}.$$  

Each of these strategies assumes that the relevant distal variables, including current distance to the object ($z$), the observer’s current velocity ($v$), and distal size of the object ($r$), as well as current visual angle of the object ($\theta$) and current time-to-contact ($\tau$; derivations are in the Appendix). Because these strategies compute the constant deceleration required to stop at the object, they would yield deceleration profiles similar to those of Strategy 1a, holding $\tau$ constant at $-0.5$, which yields the same constant deceleration. However, in contrast to the tau-dot strategy, they all require information other than optical expansion, and thus they can be tested by selectively removing information about distance, size, and velocity. In addition, because the required deceleration is computed in one step, they presumably would not yield closed-loop behavior, although there could be small final corrections.

**Strategy 4: Impulsive Braking**

There are several other strategies in which deceleration is not continuously controlled, but the brake is used in an approximate, impulsive fashion. For instance, the observer could approach the object at a constant velocity and then apply maximum deceleration late in the approach. We call this the “slam on the brakes” strategy, which results in short, fast approaches. Alternatively, the observer could apply a large deceleration early in the approach, release the brake, and slowly drift toward the target, using one or more deceleration spikes later to stop. We call this the “bang-bang” strategy, which tends to yield long, slow approaches.
strategies would yield highly variable stopping distances or times and are unlikely in actual driving because of the extreme inertial or temporal consequences. However, they could be encouraged by our use of purely visual simulations without vestibular and somatosensory information or temporal constraints. We chose to focus our analysis on trials in which braking was under ongoing visual control, and thus our conclusions only generalize to cases of continuously regulated braking.

Evidence

There are some potential problems with the tau-dot strategy. The first is that tau itself may only be detected in limited circumstances, partially due to thresholds for optical expansion (Paulus, Straube, Krafczyk, & Brandt, 1989). Humans appear to make accurate time-to-contact judgments only when contact is less than 2–3 s away (McLeod & Ross, 1983; Schiff & Detwiler, 1979; Schiff & Oldak, 1990), and this would only be exacerbated for tau-dot. However, the apparent underestimates at longer contact times could be due to the mental extrapolation task used in these experiments.

Second, Nakayama (1985) raised the possibility that the visual system does not provide a high enough temporal resolution to accurately detect second-order variables such as acceleration and tau-dot. Consonant with this observation, there is experimental evidence that the ability to detect target acceleration is very poor. Thresholds for detection of acceleration in a single target moving in the frontal plane are extremely high, with velocity ratios and true Weber fractions ($\Delta v/v$) ranging from 0.6 to 1.0 (Calderone & Kaiser, 1989; Schmerler, 1976). It is plausible that special mechanisms evolved to extract optical expansion (Frost, 1992) and would yield greater sensitivity to tau-dot than acceleration in the frontal plane. But Simpson (1991) reported very poor judgments of whether a random-dot surface approaching with constant $\tau$ values in the range around $-0.5$ would "hit" the observer. However, this task may not provide an adequate assessment, because none of these values would actually result in a "hit" if tau-dot is held constant. Glünder and Wagner (1992) similarly reported poor judgments of whether an approaching textured surface was accelerating ($\dot{\tau} < -1$) or decelerating ($\dot{\tau} > -1$). But observers may be better at determining behaviorally relevant properties such as a crash state than physical variables such as acceleration in depth.

On the other hand, there is some evidence that observers may be sensitive to tau-dot. Todd’s (1981) observers detected very small differences in time-to-contact between two approaching objects, with a threshold $\Delta \tau \tau \approx 0.01$. However, sensitivity to qualitative velocity differences is known to be an order of magnitude better than sensitivity to continuous acceleration (McKee, 1981). Lee (1976) reported that one of Spurr’s (1969) test drivers had a deceleration profile that closely fit a constant $\tau$ value of $-0.425$, although others apparently did not. More recently, Lee, Reddish, and Rand (1991) found that hummingbirds decelerate in the last 100 ms of approach to a feeding tube with a mean tau-dot value of $-0.71$, consistent with entering the tube. Similarly, Wann, Edgar, and Blair (1993) reported that adults performing several approach tasks decelerated in a manner consistent with a constant $\tau$ of $-0.45$ to $-0.5$ early in approach, but appeared to switch to another strategy during the final 300 ms. However, these were observational studies that did not dissociate tau-dot from other available information.

Kim, Turvey, and Carello (1993) isolated tau-dot by using computer displays of an approaching surface at different constant tau-dot values and asked observers to judge whether the simulated collision would be “hard” or “soft.” Contrary to Simpson’s results, observers showed a fairly steep category boundary at $\tau = -0.5$. Because holding $\tau$ constant at any value greater than $-1$ will technically result in a successful stop, Kaiser and Phatak (1993) argued that the distinction between hard and soft collisions is invalid. However, given that infinite decelerations are physically impossible, a value of $-0.5$ does specify a pragmatic boundary between safe states and crash states, and Kim et al.’s observers appear to judge this boundary reliably. Despite these suggestive results, the hypothesis that tau-dot is actually used in the control of braking has not received a direct test.

The Experiment

The present experiment was designed to test the three major types of hypotheses. Closed-loop displays of approaches to a “road sign” object were generated by computer, with the simulated deceleration determined by a spring-loaded mouse “brake” (see Figure 4). The characteristics of the brake were comparable to those of an actual automobile brake (linear, 0–0.7 g), and initial velocities (20–40 mph) and times-to-contact (3–5 s) were such that successful braking could not be performed by slamming on the brakes very late in an approach. This was intended to encourage active visual regulation of braking over the course of a trial.

The displays were manipulated to present certain combinations of optical variables. It is not possible to eliminate variables related to optical expansion ($\tau$, $\tau$, $\theta$, $\theta$) and retain a realistic approach to an object, but spatial properties ($z$, $r$, $v$) can be manipulated. Information about distance, size, and velocity can be provided by presenting objects on a textured ground surface, which specifies these properties in units of both eye height and texture scale (Gibson, 1979; Sedgwick, 1980); such information can be eliminated by presenting objects hanging in empty space. Dependence on object size can also be tested by keeping distal size constant or varying it randomly between trials. The two factors of a present or absent ground surface and constant or variable object size yielded four conditions:

1. Air-variable size: This condition provides only $\tau$, $\tau$, $\theta$, and $\theta$. Displays simulate an approach to an object in empty space, and object size is randomized. This isolates optical expansion variables, eliminating information for distance, size, and velocity. Reliable performance in this
condition would support the $\hat{r}$ and $\theta$ strategies, which can be distinguished by their braking profiles.

2. Air–constant size: This condition provides $\tau$, $\tau$, $\theta$, $\theta$, and $r$. This is the same as Condition 1, except that distal object size is kept constant. A constant size object could make a size-based strategy more reliable.

3. Ground–variable size: This condition provides $\hat{r}$, $\tau$, $\theta$, $\theta$, $r$, $z$, and $v$. Displays add a ground surface with a horizon, providing information about distance, velocity, and size in units of eye height and texture width but randomize distal size between trials. A significant improvement in performance over Condition 1 would support one of the computational strategies.

4. Ground–constant size: This condition provides $\hat{r}$, $\tau$, $\theta$, $\theta$, $r$, $z$, and $v$. This is the same as Condition 3, except that distal size is held constant. A significant improvement over both Conditions 2 and 3 would support the use of a size-based strategy.

It should be noted that these braking strategies are not mutually exclusive. It is conceivable that an observer might use different strategies depending on the available information or other factors such as the strength of the brake or time-to-contact. This may be assessed by relative performance across the four conditions. Successful braking with the predicted deceleration profile in Condition 1 would demonstrate that a tau-dot strategy is sufficient, if not necessary. If performance improves or braking profiles change with additional information about distance, size, and velocity, this would demonstrate a contextual dependence on spatial variables. If performance and braking profiles are similar with and without spatial information, this would strongly suggest a general reliance on a tau-dot strategy.

**Method**

**Participants**

Data were collected from 15 observers, but three were eliminated because they did not exhibit the continuous regulation of braking that was the subject of the experiment. Two of them used a bang-bang strategy on nearly every trial, resulting in long protracted approaches, and one used a slam strategy on nearly every trial. The remaining 12 participants ranged in age from 20 to 51 years (only 2 were over 30) and included 9 men and 3 women. One participant (male, 20) had never driven a car, and the rest had at least 3 years of driving experience; all were right-handed and had normal or corrected-to-normal vision. Apart from the two authors,
the other participants were naive as to the purpose of the experiment.

**Displays and Apparatus**

Real-time displays simulating an approach to three diamond-shaped “road signs” (Figure 4) were generated with a Silicon Graphics (Mountain View, CA) IRIS 4D/210 GTX workstation and presented on a raster monitor with a refresh rate of 60 Hz. Displays had a pixel resolution of 1,280 H × 1,024 V and were presented at a rate of 30 frames/s. The screen subtended a visual angle of 40° H × 32° V and was viewed binocularly from a chin rest at a distance of 43 cm. A matte black viewing box was placed in front of the monitor, with the screen visible through a window at one end and the chin rest at the other end. It should be noted that although the black edges of the window were not subjectively prominent against the black background of the display, they were visible. The window may thus have provided a fixed frame of reference, like the windshield in a car.

The “brake” consisted of an optical mouse that moved in a slide attached to a spring-loaded handle, with dynamics modeled on an automobile brake. Resistance was provided by a linear spring with a spring constant of 138.3 N/m; minimum force required to displace the mouse was 2.5 N, and maximum force was 21.8 N at full extension, with a total excursion of 14 cm. Due to the spring’s linearity, changes in the position of the mouse were proportional to changes in the amount of force applied. The position of the mouse was converted directly into simulated deceleration by the computer, preserving the linearity of the relationship between applied force and deceleration. Automobile brakes are similarly linear over most of their range, with a nonlinear upturn near full depression. The simulated deceleration ranged from 0 g in the 0 cm position to 0.7 g in the 14 cm position, corresponding to the upper end of the linear portion of a typical automobile brake (G. Schaefer, General Motors, personal communication, November 1990). The mouse could only be used to increase or decrease the deceleration; there was no way to accelerate. The mouse pad had a resolution of 1,024 pixels within the 14-cm excursion, allowing precise control of deceleration. The position of the mouse was sampled at 30 Hz, with a display loop time of 1/30 s. The apparatus was mounted on a table so that the participant could comfortably rest his or her right hand on the handle; the position of the handle was 40 cm to the right of the display, and initially 12 cm forward of the eye position.

Displays were generated in the four conditions described earlier: air-variable size, air-constant size, ground-variable size, and ground-constant size. We assumed an eye height of 1.1 m as a distance metric, arrived at by measuring the seated eye height of a person with a 1.6 m standing eye height as he sat in the driver seat of several different cars. Initial conditions were varied randomly between trials: Five different starting distances (e) were crossed with five different initial velocities (v) ranging from 8.0 to 15.0 e/s. This structure provided three scales of nested texture that continued to expand even after the large object had filled the screen, making τ information available at close viewing distances. The diamond orientation minimized pixel creep. The centers of the large diamonds were separated by 1.2 times their width, and the gap between the small diamonds was equal to one-third of their width. In the two constant size conditions, the width of each road sign was 1.2 e from corner to corner, for a total width of 4.1 e across the three objects. In the two variable size conditions, a different object width was selected randomly on each trial from a range of 0.6 to 1.8 e, for a total width of 2.0 to 6.1 e.

In the two ground conditions, the ground surface was a gray and black checkerboard 1.0 e below eye level. Each square measured 1.0 e on a side; the whole surface was 6.0 e (6 squares) wide and extended to a pseudohorizon 55.5 e from the station point. A blue “sign post” anchored each object to the surface at a point directly beneath it, specifying its position on the ground surface. Post width was one-fifteenth of the sign’s width and varied with object size. In the two air conditions, the ground surface and sign posts were absent, but the displays were otherwise identical to those in the ground conditions.

**Procedure**

Displays were blocked by condition and presented in two 1-hour sessions in a within-subject design. The two air conditions were presented in one session and the two ground conditions in the other session in a counterbalanced order; each condition contained 100 trials. At the beginning of each session, participants received an additional 100 practice trials in the corresponding surface condition with a variable size object, so they could become familiar with the characteristics of the brake.

The first 9 participants were instructed to try to stop “as close as possible” to the objects. To encourage continuous regulation of braking, they were also told that “the brake was not powerful enough to stop the motion if it was used only at the last moment,” so they would “usually need to start braking right away, and then adjust the brake in such a way as to get close to the object.” For the last 6 participants, performance was also monitored during the first 50 practice trials to ensure that they were actively braking. Participants who used slam-on-the-brakes or bang-bang strategies excessively were encouraged to “try to brake as if you were really driving this car.” Those using the bang-bang strategy were also told that “the approach should feel as comfortable as possible; try to use the brake more smoothly. Pumping the brake that hard would feel uncomfortable in a real car.” Those using the slam-on-the-brakes strategy were told to “try to use the brake more at the beginning,” because it would “make crashing less likely in the end” and would “feel more comfortable in a real car.” After such monitoring, they used a continuously regulated strategy most of the time.

A participant initiated a trial by pressing the space bar with the left hand. This caused the intertrial screen (a uniform light blue) to go black, and after a 1-s pause, the display began. Displays ended after the participant either came to a stop or crashed into the object. If velocity was successfully brought to zero before collision, the final frame of the trial was displayed for 1 s longer, followed by the intertrial screen. If the participant crashed, the central blue diamond would fill the screen and the final frame would be displayed for 1 s while a high-pitched tone sounded. This was followed by the intertrial screen.

**Analysis**

The raw data from a trial consisted of time series of the participant’s simulated distance from the object (z), velocity (v), and deceleration (d). The τ and in every frame were then calculated from z, v, and d, following Lee (1976):
Figure 5. Sample time series of individual trials. Panel a: A regulated trial resulting in a safe stop, with mean $\tau = -0.53$, $r = -0.98$. Panel b: A regulated trial resulting in a crash, with mean $\tau = -0.74$, $r = -0.99$. Window borders and regression line represented by dotted lines; + denotes start point with increase in deceleration, — denotes start point with decrease in deceleration, * denotes stop point; $e$ = eye height.
\[ \tau = z/v \]  
\[ \tau = -1 + \left( \frac{z}{v^2} \right). \]

Representative time series for \( d, \tau, \) and \( r \) appear in Figures 5a and 5b. The time-to-contact at the onset of braking \( (\tau_r) \) was determined as the \( \tau \) value in the first frame with a nonzero deceleration. The final stopping distance \( (z_r) \) between the observer and the object was determined from the final frame on successful trials; on crash trials, it was extrapolated from the velocity and deceleration in the final frame, yielding a negative distance (i.e., behind the object). Notice that when the observer stops short of the object (Figure 5a), as velocity goes to zero, both time-to-contact and tau-dot go to infinity. Conversely, in a crash (Figure 5b), as distance goes to zero, tau-dot goes to \(-1\). These are mathematical consequences of Equation A5 (see the Appendix).

**Trial classification.** Of the 4,800 test trials collected, 26 were discarded because the final deceleration was virtually zero, yielding nearly infinite stopping distances behind the object. The remaining 4,774 trials were sorted into three classes based on visual inspection of the deceleration time series. Slam-on-the-brakes trials (6% of all trials) were characterized by a quick increase from zero to a large deceleration value (often the maximum), usually late in the approach, which was not adjusted further. Bang-bang trials (11%) were characterized by a positive deceleration peak often early in the approach, which was then released completely for a substantial portion of the trial as the brake was moved back to the 0-cm position, followed by one or more separate deceleration spikes late in the approach. In regulated trials (83%), deceleration was applied at some point and then maintained at a fluctuating positive value throughout the rest of the trial. Usually, several consecutive increases and decreases occurred (Figure 5a), but trials showing only an increase were differentiated from slam-on-the-brakes trials if this increase was gradual over time (Figure 5b), rather than abrupt and uniform. Only regulated trials were analyzed further.

**Estimating \( \tau \).** We used two different methods to estimate the margin value of tau-dot used by participants, which we will call the mean tau-dot and the critical tau-dot. To obtain the mean \( \tau \), we first windowed the time series by hand to isolate the continuously regulated portion of the trial and then estimated the average rate of change in \( \tau \) (i.e., \( \dot{\tau} \)) by computing the slope of the regression line. Sample windows and regression lines are indicated by dotted lines in Figures 5a and 5b. Windowing eliminated the initial rise in \( \tau \) from \(-1\) (no deceleration) to the first plateau, which corresponded to putting on the brakes to reach the first regulated value of deceleration. The beginning of the window was chosen on the basis of the deceleration plot, at the end of the first increase in deceleration. Windowing also eliminated the final rise in tau-dot to an infinite value on successful trials, or the final tailing off to \(-1\) on crash trials, which are mathematical consequences of stopping in front of or crashing into the object. The end of the window was thus chosen on the basis of the tau-dot plot, at the onset of these final excursions. Within the window, we performed a linear regression of \( \tau \) on time, using \( |r| = .6 \) as a criterion value for an adequate fit. The resulting slope is the mean tau-dot value for the regulated portion of the trial. A total of 94% of the regulated trials had correlations of \( |r| \geq .6 \), leaving 3,721 trials for the estimate of mean tau-dot; of these, 94% had fits with \( |r| > .8 \). For example, the successful stop in Figure 5a had a mean \( \tau = -0.53, r = -0.98 \), and the crash in Figure 5b had a mean \( \tau = -0.74, r = -0.99 \).

The second measure, the critical value of \( \tau \), was determined in two ways from a detailed analysis of brake adjustments. Suppose braking is regulated with respect to a margin value \( \tau_m = -0.5 \). If \( \tau \) is above \(-0.5 \) at a given moment, it specifies that the current deceleration is too high, and we would expect the participant to pull back on the brake. Conversely, if the current \( \tau \) is below \(-0.5 \), this specifies that deceleration is too low, and we would expect the participant to push the brake forward. To analyze the direction of each brake adjustment, we wrote a computer program that identified transition points in the deceleration time series. Start points occurred when the participant started moving the brake, acting either to increase \((+)\) or decrease \((-) \) the level of deceleration (see labels in Figure 5); stop points occurred when the participant stopped moving the brake; and reversal points occurred when the motion of the brake abruptly reversed direction (i.e., a combined stop and start). The program first identified plateaus during which deceleration was held at a constant value \((\geq 0.0028 \text{ g})\) for at least 200 ms and found start and stop points by identifying the direction of brake movement at the onset and offset of the plateau. It then located regions of the deceleration trace in which a U-shaped pattern occurred within 200 ms, and identified their extremum as reversal points. The 200-ms cutoff was chosen as a liberal estimate of reaction time, to ensure that only distinct adjustments of the brake would be counted as start and stop points. Of the 16,261 points identified, 42% were starts, 14% were reversals, and 44% were stops.

To determine the critical tau-dot in each condition, we plotted the percentage of adjustments that were positive (i.e., acted to increase deceleration) as a function of the current tau-dot value, binning the data in \( T \) intervals of 0.05. We then fit each participant’s data with an ogive by performing a \( z \) transformation on the percentage of adjustments and computing a linear regression on \( T \). The tau-dot value at which the regression line crossed 50% positive adjustments was interpreted as the critical value, because below this value observers tended to increase deceleration, whereas above it they reduced deceleration. The tau-dot value at which the gression line crossed 75% was used to determine a perceptual-motor difference threshold (just noticeable difference, or JND, in \( \tau \) and tau-dot ratio (JND/\( \tau \)). This provides an estimate of the difference between the current value and margin value of tau-dot \((\tau_m - \tau)\) that elicited an active brake adjustment half the time; a purely visual JND could be much smaller. Note that this estimate is only valid at the critical value, not for other parts of the tau-dot range, and thus is not a true Weber fraction. Two of the 12 participants almost always increased deceleration and thus their data could not be fit to obtain a critical tau-dot. For six of the remaining 40 cells, the number of adjustments was insufficient for accurate estimates. We did not break this data down by initial time-to-contact because of the shortage of data.

**Magnitude of adjustment.** To determine whether the magnitude of brake adjustments was visually regulated, we first identified consecutive transition points that corresponded to the onset and offset of brake movement. In particular, a start point followed by a stop point, a reversal followed by a stop point, or a start point followed by a reversal specified a unitary brake movement. The magnitude of adjustment is simply the difference between the initial and final values of deceleration \((\Delta \dot{\tau})\) at the onset and offset of a movement, or alternatively, the difference between the initial and final values of tau-dot \((\Delta \tau)\). A total of 6,253 such adjustments were identified in the data. To assess whether the amount of braking depends on the current value of tau-dot, we plotted the magnitude of adjustment (in terms of both \( \Delta \dot{\tau} \) and \( \Delta \tau \)) as a function of the \( \tau \) value at the onset of the adjustment, binning the data in \( T \) intervals of 0.05, and performed a linear regression. The slope of the regression line describes the observer’s calibration of adjustment magnitude to the current value of tau-dot. In addition, the tau-dot value at which the regression line crosses an adjustment
magnitude of zero provides a converging measure of the critical tau-dot. Due to the binning procedure, there were insufficient data for a reliable estimate of braking magnitude in 40 of the 504 cells.

Finally, to evaluate the \( \tau \) hypothesis, we measured the drift in \( \tau \) during each plateau, when the brake was not moving and deceleration was constant. A plateau was specified by a stop point followed by a start point, and we eliminated those in which deceleration was zero (\( \tau = -1 \)) or at ceiling. The tau-dot drift is simply the difference between the initial and final tau-dot values at the onset and offset of the plateau. A total of 6,414 such plateaus were identified in the data. If observers detect a drift in tau-dot in order to initiate an adjustment, we would expect a systematic relationship between the current value of tau-dot and the amount of drift prior to the next adjustment (such as a constant threshold or proportional relation). We thus plotted \( \tau \) drift as a function of the \( \tau \) value at the onset of the adjustment, binning the data in initial \( \tau \) intervals of 0.05.

Results

Preliminaries

Of the 4,774 trials, the vast majority (83%) were classified as regulated trials, 11% as bang-bang trials, and 6% as slam-on-the-brakes trials (Figure 6). There appeared to be a slight increase in nonregulated strategies with longer initial times-to-contact: with \( \tau_0 = 3.0 \) s, 86% of the trials were regulated, whereas with \( \tau_0 = 5.0 \) s, 74% were regulated. Most important, spatial information did not influence strategy choice, for the percentage of regulated trials was constant across conditions: 82% in air-variable size, 84% in air-constant size, 84% in ground-variable size, and 81% in ground-constant size.

The time series in Figure 5a is an example of a regulated trial in which the initial deceleration was too great, resulting in a \( \tau \) value slightly above \(-0.5\). The deceleration was then reduced to an inadequate level, yielding a \( \tau \) slightly below \(-0.5\); subsequently increased once more; and finally increased again to stop in front of the object (thus \( \tau \) rises to infinity). Note that the deceleration is applied, held constant for a time, and then adjusted, with each brake adjustment moving \( \tau \) toward \(-0.5\). This pattern of oscillation between plateaus is typical and is characteristic of closed-loop control rather than computation of a constant deceleration value. The mean time length of these deceleration plateaus across the whole data set was \( 0.71 \pm 0.61 \) s.

Figure 5b depicts a trial in which deceleration was increased to progressively higher plateaus. The deceleration varies greatly over the course of the trial, but the resultant \( \tau \) hovers between \(-0.6 \) and \(-0.8\), and thus the trial ends in a crash (with \( \tau \) dropping to \(-1\)). Nevertheless, each brake adjustment moves \( \tau \) toward a value of \(-0.5\). This trial also illustrates the fact that during each deceleration plateau, the \( \tau \) value drifts away from \(-0.5\) (unless \( \tau = -0.5 \)).

Success Measures

Three measures of the success of braking performance were evaluated, including constant error and variable error in stopping distance and the percentage of crashes.

Constant error in stopping distance. Mean signed stopping distance (\( z_s \)), or constant error, is plotted as a function of initial time-to-contact (\( \tau_0 \)) for each condition in Figure 7. Overall, mean stopping distance had a positive constant error of 1.7 eye heights in front of the object, with closer stops in the air conditions on the shortest trials (\( \tau_0 = 3 \) s), indicative of more crashes. An analysis of variance (ANOVA) on stopping distance revealed no main effect of

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![Figure 6](image_url)  
*Figure 6.* Percentage of trials in each strategy classification, by initial time-to-contact (\( \tau_0 \)) and condition. 1 = air-variable size, 2 = air-constant size, 3 = ground-variable size, 4 = ground-constant size.
condition, $F(3, 33) = 1.34, ns$, but a main effect of initial time-to-contact, $F(4, 44) = 9.95, p < .001$, accounting for 25% of the total sum of squares, and a significant interaction, $F(12, 132) = 4.73, p < .001, 7.8\%$ of the total SS. Tukey post hoc tests showed that the only significant differences were between the air conditions and the ground conditions at $\tau_0 = 3$ s, $HSD = 1.029, p < .01$. Thus, the air conditions yielded closer stops only on short high-velocity trials.

Variable error in stopping distance. The mean standard deviation in stopping distance, or variable error, is plotted as a function of initial time-to-contact in Figure 8. The SDs were computed for each participant individually and then averaged. An ANOVA revealed no main effect of condition, $F(3, 33) = 1.35, ns$, but a main effect of initial time-to-contact, $F(4, 44) = 15.9, p < .001, 14\%$ of the total SS, and an interaction, $F(12, 132) = 2.59, p < .01, 6.6\%$ of the total SS. Tukey tests showed that the only significant differences occurred at $\tau_0 = 3$ s, $HSD = 1.029, p < .01$. Thus, the air conditions yielded closer stops only on short high-velocity trials.

Percentage of crashes. The percentage of crashes in each condition is plotted as a function of initial time-to-contact in Figure 9. There is an increase in the percentage of crashes at short $\tau_0$ values, mirroring the stopping distance data (Figure 7). The percentage of crashes is 20% higher in the air conditions than the ground conditions in short high-velocity trials ($\tau_0 = 3$ s).

Taken together, the three success measures show a clear pattern across conditions. At the shortest initial time-to-contact, the air conditions yield riskier stops, more crashes, and greater variability. Thus, when there is only 3 s to initiate braking, the presence of a ground surface provides an advantage. Otherwise, performance is comparable when optical expansion alone is available.

Time-to-Contact at the Onset of Braking

Figure 10 depicts the time-to-contact at the moment the brakes were first applied ($\tau_b$), as a function of initial time-to-contact at the start of the trial ($\tau_0$). The shaded region corresponds to $\pm 1 SE$ from the overall mean and represents the mean within-subject standard error, computed separately for each participant in each condition. An ANOVA revealed no main effect of condition, $F(3, 33) = 0.279, ns$, or an interaction, $F(12, 132) = 1.342, ns$; however, a main effect of $\tau_0$ was found, $F(4, 44) = 48.6, p < .001, 67\%$ of the total SS.

It is not surprising that $\tau_b$ decreases with shorter $\tau_0$, given that there is less time to brake. A linear regression on the condition means yielded $\tau_b = 0.64 + 0.56\tau_0$, with $r = .99$. Because the slope is less than 1, it is clear that participants are not simply braking a constant time after trial onset. Note that with $\tau_0 = 3$ s, braking is initiated after 0.68 s, leaving very little time to bring the car to a stop and pushing the upper limit of braking power. This could account for the closer stopping distance and higher percentage of crashes in these trials. Most important, the lack of condition effects

![Figure 7](image1.png)

*Figure 7. Mean constant error in stopping distance as a function of initial time-to-contact ($\tau_0$). Positive values indicate stops in front of the object. Air, Var = air variable; Air, Const = air constant; Gnd, Var = ground variable; Gnd, Const = ground constant.*

![Figure 8](image2.png)

*Figure 8. Mean variable error in stopping distance as a function of initial time to contact ($\tau_0$); that is, the mean of each participant’s standard deviation in stopping distance. Air, Var = air variable; Air, Const = air constant; Gnd, Var = ground variable; Gnd, Const = ground constant.*
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Figure 9. Mean percentage of trials in which a crash occurred, as a function of initial time-to-contact. Air, Var = air variable; Air, Const = air constant; Gnd, Var = ground variable; Gnd, Const = ground constant.

implies that the timing of the onset of braking does not depend on ground surface information.

\( \dot{\tau} \) Estimates

Mean \( \dot{\tau} \). The overall mean tau-dot in each condition is plotted as a function of initial time-to-contact in Figure 11, and values for each participant are given in Table 1. Mean tau-dot was nearly identical in each condition, with a grand mean of \( \dot{\tau} = -0.51 \) (SD = 0.22 across all trials), precisely as predicted by the tau-dot hypothesis. An ANOVA revealed no main effect of condition, \( F(3, 33) = 1.46, \ ns \); or a main effect of \( \tau_0 \), \( F(4, 44) = 1.94, \ ns \); but a significant interaction, \( F(12, 132) = 3.00, p < .001 \), accounting for 8.8% of the total SS. Tukey post hoc tests showed that the only difference occurred at \( \tau_0 = 3 \) s between the air conditions (\( M = -0.54 \)) and the ground conditions (\( M = -0.43 \)), \( HSD = 0.094, p < .01 \). Thus, mean tau-dot is more conservative in the ground conditions only on the shortest high-velocity trials. Otherwise, there was no effect of the ground surface or fixed object size, indicating that optical expansion is not only sufficient for successful braking, but that braking strategy appears to be consistent across conditions. There were individual differences in mean tau-dot, with subject means ranging from \( \dot{\tau} = -0.35 \) to \( -0.61 \) (SD = 0.086; see Table 1).

Whereas the observed mean of \( \dot{\tau} = -0.51 \) was closely predicted by the tau-dot strategy, it was far from the value of +1.0 predicted by the rate of expansion (\( \dot{\theta} \)) strategy. The time series of \( \dot{\theta} \) (not shown) systematically increased over the course of a trial, reached a peak, and then declined, rather than remaining near any constant value \( k \). The increase in \( \dot{\theta} \) was large, such that fluctuations due to brake adjustments were minor by comparison. These results are contrary to the \( \theta \) strategy.

Critical value of \( \dot{\tau} \). The percentage of start, stop, and reversal adjustments that increased deceleration are plotted as a function of the tau-dot value at the onset of the adjustment for each condition in Figure 12a. The pattern of start and reversal points was highly similar, but stop points did not show a similar pattern nor were they systematically related to a specific tau-dot margin value. In fact the curve for stop points was nearly flat for most of its range (it might have been completely flat if acceleration as well as deceleration had been allowed). Subjects do not seem to be regulating the stopping of brake movements using \( \dot{\tau} \) (for instance, moving the brake until they see \( \tau = -0.5 \)). If they were, we would expect a curve similar to those for the start and reversal adjustments, whose direction does appear to be related to the \( \dot{\tau} \) value at movement initiation.

We thus combined start and reversal data for analysis, plotted by condition in Figure 12b. Overall, the mean critical value \( \tau \) was \( -0.44 \), close to the predicted value of \( -0.5 \). Condition means were nearly identical: \( -0.41 \) in air-variable size, \( -0.45 \) in air–constant size, \( -0.46 \) in ground–variable size, and \( -0.44 \) in ground–constant size, \( F(3, 27) = 0.79, \ ns \). There were also individual differences in critical \( \dot{\tau} \) with subject means ranging from \( -0.31 \) to \( -0.52 \) (SD = 0.067).

The perceptual-motor difference threshold (JND in \( \dot{\tau} \)) was 0.24 overall, corresponding to the \( \tau_m - \dot{\tau} \) difference that

Figure 10. Mean time-to-contact at the onset of braking (\( \tau_m \)) as a function of initial time-to-contact (\( \tau_0 \)). Shaded area represents \( \pm 1 \) SE about the overall mean, the mean within-subject SE computed separately for each participant in each condition. Air, Var = air variable; Air, Const = air constant; Gnd, Var = ground variable; Gnd, Const = ground constant.
elicited a brake adjustment. By condition, the thresholds were 0.25 in the air-variable size condition, 0.24 in the air-constant size condition, 0.22 in the ground-variable size condition, and 0.24 in the ground-constant size condition, F(3, 27) = 0.74, ns. Thresholds for individual participants ranged from 0.15 to 0.33 (SD = 0.061). The corresponding tau-dot ratio is 0.24/0.44 = 0.55.

Magnitude of Adjustment

The magnitude of brake adjustment (in units of the resulting change in tau-dot, Δτ) is plotted as a function of the tau-dot value at the onset of adjustment in Figure 13; data from all four conditions were grouped together for the plot, as they showed virtually identical patterns. An ANOVA on Δτ revealed a main effect of tau-dot at onset, F(20, 220) = 48.8, p < .001, but neither a main effect of the air versus ground conditions, F(1, 11) = 0.05, ns, nor an interaction, F(20, 220) = 1.44, ns.

As can be seen, the magnitude of adjustment increases as tau-dot departs from -0.5. A linear regression yielded the equation Δτ = -0.54 -1.04T, with r = .98. First, the slope of this line is exactly -1, although individual slopes ranged from -1.63 to -0.59. This indicates that observers not only move the brake in the direction of τm, but that they do so by an amount that on average brings τ back to τm. Second, the tau-dot value at which the regression line crosses an adjustment magnitude of zero provides another measure of critical tau-dot, resulting in a value of -0.52, again close to the theoretical value of -0.5. This data is consistent with the theory that brake adjustments are on average directly proportional to τm - τ, with τm ≈ -0.5.

Representing adjustment magnitude in units of change in deceleration (Δd) yields a regression equation of Δd = -2.42 -4.32τ, with r = .99. Individual slopes ranged from -7.66 to -1.59. An ANOVA on Δd revealed a main effect of tau-dot at onset, F(20, 220) = 43.8, p < .001, but no main effect of air versus ground condition, F(1, 11) = 1.32, ns, and no interaction, F(20, 220) = 0.84, ns. Again, there appears to be a linear relationship between adjustment magnitude and tau-dot at the onset of the adjustment. In sum, the results support the theory that tau-dot is used to determine both the direction and magnitude of brake adjustment, with or without a ground surface.

Table 1

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<th>SD in mean τ</th>
<th>Critical τ (direction)</th>
<th>JND in τ</th>
<th>Critical τ (magnitude)</th>
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Note. Critical tau-dot could not be estimated for two participants because they made very few negative adjustments. M = male; F = female; JND = just noticeable difference.

* The one nondriver in the sample.
Figure 12. Percentage of brake adjustments that increase deceleration, as a function of T at the onset of the adjustment. Panel a: Data for start, reversal, and stop points. Panel b: Combined data for start and reversal points, by condition. Critical T values for each condition are in parentheses. Air, Var = air variable; Air, Const = air constant; Gnd, Var = ground variable; Gnd, Const = ground constant.

Discussion

Braking Strategies

The results strongly support the hypothesis that a tau-dot strategy is sufficient for the visual control of braking, although the following conclusions are limited to continuously regulated trials. Participants successfully performed the task in the air-variable size condition when only the optical expansion information was available (T and 0). Our empirical estimates of Tm yielded a mean tau-dot of -0.51, a critical tau-dot of -0.44 based on direction of adjustment, and a critical tau-dot of -0.52 based on magnitude of adjustment. These are all very close to the theoretical value of -0.5 predicted by Strategy 1 (T), but much lower than the value of +1.0 predicted by Strategy 2 (θ). Thus, barring an alternative hypothesis we have not considered, the results clearly indicate that observers can use a tau-dot strategy to control braking, at least in the absence of other spatial information.

Further, the results imply that observers use a tau-dot strategy even when spatial information is available. There were no differences in our measures of braking success, tau-dot, or adjustment magnitude when a ground surface was added or object size was fixed (with the exception of short, T0 = 3 s, trials, which we consider below). The strong similarity of braking behavior with and without a ground surface suggests that participants rely on tau-dot under all conditions, even when information about distance, size, and velocity is available. However, computing a constant deceleration from spatial variables (Strategy 3) would also yield a mean T = -0.5 and brake adjustments proportional to Tm - T. It is thus logically possible that observers use a tau-dot strategy in air conditions and a spatial strategy in ground conditions. On the other hand, the time series clearly indicate that deceleration is not computed in one step with final corrections, but that braking is an ongoing closed-loop behavior that involves successive approximations of Tm. In addition, the fact that different observers have different Tm values cannot be explained by computing a constant deceleration, which would yield a mean value of -0.5. We thus believe that the results militate against Strategy 3.
Moreover, the data implicate a specific tau-dot strategy. First, we can rule out Strategy 1a, the hypothesis that tau-dot is held constant at \( \tau_m = -0.5 \). As others have emphasized (Kaiser & Phatak, 1993), only rarely do braking records show constant tau-dot approaches, and there is a great deal of variability in the time series from trial to trial. A typical pattern in our data is that deceleration is adjusted and then kept constant for a plateau period on the order of one-half second or more (Figure 5), as though the observer is adjusting the brake, detecting the optical consequences, and then readjusting the brake accordingly. Thus we can eliminate a strict version of the tau-dot strategy in which \( \tau \) is held constant. However, just because tau-dot is not held constant does not mean it is not used as a control variable; indeed, oscillation about a critical value is a signature of a regulated variable. One possibility is a closed-loop version of this strategy in which observers monitor tau-dot continuously during an adjustment until a set-point of \(-0.5\) is reached. However, the observed stop points were not systematically related to such a margin value; many adjustments occur quickly and overshoot or undershoot the margin value. This suggests a discrete, ballistic adjustment strategy.

We can also rule out Strategy 1d, in which the direction of adjustment is based on the direction of drift in \( \tau \). First, at reversal points, brake adjustments are made after little or no drift in tau-dot, yet the pattern of adjustment is virtually the same as it is for start points at the ends of plateaus. In contrast, reversal points could result from detecting \( \tau \) per se. Second, this strategy cannot account for individual differences in \( \tau_m \), for under constant deceleration the direction of tau-dot drift always changes sign at \( \tau = -0.5 \). Third, if subjects were detecting drift in tau-dot, we would expect the amount of drift prior to an adjustment to be constant or proportional to the current tau-dot and to bear some relation to the JND in tau-dot. Yet neither the mean drift nor its variability is systematically related to tau-dot (Figure 14). Thus, the variable of interest appears to be tau-dot per se, not its rate of change.

We propose that the current value of tau-dot is used to determine both the direction and approximate magnitude of the next brake adjustment. It is clear from Figure 12 that the direction of adjustment is strongly related to current tau-dot, in that participants tend to increase deceleration when \( \tau < -0.44 \), and reduce deceleration when \( \tau > -0.44 \), consistent with Strategy 1b. But the control relation appears stronger than this, for the magnitude of adjustment also depends on current tau-dot. This is a linear relationship with a slope of \(-1\), such that brake adjustments are on average equal to the difference between the current value of tau-dot and a margin value of \(-0.52\), consistent with Strategy 1c: \( \Delta \tau = (\tau_m - \tau) \). This tends to bring \( \tau \) back to \(-0.52\) and keeps the required deceleration within practical bounds. Of course, because this is only true on average, there are frequent undershoots and overshoots, yielding closed-loop behavior with succes-

![Figure 13](image)

**Figure 13.** Magnitude of brake adjustment in \( \Delta \tau \) units, as a function of \( \tau \) at the onset of adjustment. Shaded area represents \( \pm 1 \) SE about the overall mean (the mean within-subject SE).

![Figure 14](image)

**Figure 14.** Drift in \( \tau \) during deceleration plateaus as a function of \( \tau \) at the start of the plateau. Error bars represent \( \pm 1 \) SE of all trials.
sive approximations of $\tau_m$ and variation in the stop point. A similar pattern of oscillation about $\tau_m$ can be seen in the slopes of the $\tau$ time series in Lee et al. (1991) and Wann et al. (1993), although it is more evident in the $\tau$ time series of Figure 5. In sum, the data on regulated trials are highly consistent with Strategy 1c.

The Ground Surface Advantage

The one discrepancy to be explained is the selective deterioration in the air conditions at the shortest initial time-to-contact ($\tau_0 = 3$ s). The presence of a ground surface provided an advantage in stopping accuracy, variable error, and crash avoidance on short trials, as well as a slightly higher value of mean tau-dot ($-0.43$ for ground, as opposed to $-0.54$ for air). Given that initial distance and initial $\tau_0$ were crossed in our design, the shortest $\tau_0$ was correlated with the highest initial velocities ($v_0 = \sqrt{2g/\tau_0}$). We suggest that the high velocity of the ground surface was detectable before the early expansion of the object, and participants used it as a cue to initiate "emergency" braking on short trials. In the words of one participant, in the air condition "you can't tell right away if you're really going to have to hit the brakes hard." There was no difference in the timing of braking onset between conditions. However, the magnitude of the first brake adjustment at $\tau_0 = 3$ s was different, with a mean of 0.59 in ground trials and 0.48 in air trials. Although statistically nonsignificant, this difference is suggestive. It seems that emergency braking with the ground surface on short, fast trials involved a larger initial brake adjustment, rather than braking earlier. In sum, we believe the discrepancy at $\tau_0 = 3$ s is due to an artifact of the display that participants learned to exploit. In natural braking, $v_0$ is not correlated with $\tau_0$ because there are no discrete trials with initial conditions, rather, approaches are continuous and distances are random.

Calibrating the Magnitude of Adjustment

It is interesting that given the indeterminate relationship between tau-dot and deceleration, $d = (v/\tau)(\tau + 1)$, a linear relationship holds between current $\tau$ and magnitude of adjustment whether it is measured in units of $\Delta \tau$ or $\Delta d$. We performed a correlation between $\Delta \tau$ and $\Delta d$ for all the adjustments in our data set and obtained a surprisingly high value, $r = .77$. This is a reflection of the fact that $v$ and $\tau$ tend to decrease together, such that deceleration is roughly proportional to tau-dot during most of the approach (see top panel of Figure 2). It thus seems likely that observers calibrate the amplitude of brake movement ($\Delta d$) required to produce a particular change in tau-dot ($\Delta \tau$) and make discrete anticipatory adjustments: $\Delta d \propto \Delta \tau = (\tau_m - \tau)$. Although the relationship between movement amplitude and $\Delta \tau$ breaks down near the end of the approach, particularly for $\tau < -0.5$, the nonlinear upper range of an actual car brake could act to linearize it when in a crash state.

A related issue is the apparent departure from a tau-dot strategy in the last half-second to second before stopping (Figure 5), which could suggest a two-phase braking strategy (e.g., Wann et al., 1993). Stopping short of the object necessarily sends tau-dot to infinity, whereas crashing into it necessarily sends tau-dot to $-1$. Why do not participants correct for these departures from $-0.5$, and stop directly at the object? We believe this pattern of data is consistent with the magnitude of adjustment theory, and does not require a special strategy. First, there is typically a delay of more than a half-second between adjustments (mean time length of deceleration "plateaus" was $0.71 \pm 0.61$ s). Thus there may not be time to correct for an error detected in the last half-second or so. Further, this is the nonlinear region in which the relation between brake movement and tau-dot breaks down. When stopping slightly short, the observer is indeed successfully coming to a stop before hitting the object, as instructed, and there is no reason to let up on the brake and risk overshooting. On the other hand, when going into a crash, the brake is often already at full extension and there is no braking power left (e.g., Figure 5b). Thus, we believe no special strategy is needed to account for the last second of data.

Detecting Changes in $\tau$

The mean change in tau-dot that elicited a brake adjustment was 0.24, corresponding to a tau-dot ratio of 0.24/0.44 = 0.55. (This is not a true Weber fraction because it is based on a single estimate at the critical tau-dot of $-0.44$.) In other words, a difference of $\pm 0.24$ from $\tau_m = -0.44$ elicited a brake adjustment half the time. This is not a high level of sensitivity, but it is based on a perceptual--motor measure that incorporates the detection of a visual change, the decision that the change is large enough to warrant a brake adjustment, and the perceptual--motor loop time. Purely visual JNDS are likely to be smaller. Nevertheless, the perceptual--motor JND is adequate for controlling braking and maintaining tau-dot in the neighborhood of a margin value. This does not imply, however, that the visual system detects tau-dot per se. Whether we are sensitive to $\tau$ as a continuous second-order variable or are detecting successive values of $\tau$ (a quantal $\Delta \tau$) remains an open question.

Individual Differences

Strategy 1c would also allow the observed pattern of individual differences. Although $\tau_m = -0.5$ theoretically yields the most efficient stops, higher values are more conservative, yielding slower, longer approaches, whereas lower values are more risky, yielding faster, shorter approaches and requiring a preemptive second phase of high deceleration late in the approach. Subject means of tau-dot ranged from $-0.35$ to $-0.61$, critical values estimated from the direction of brake adjustments ranged from $-0.31$ to $-0.52$, and critical values estimated from the magnitude of adjustment ranged from $-0.33$ to $-0.64$. This indicates that there may be individual differences in braking strategy, with both cautious and risky drivers, although all of them are in the range near $-0.5$. Even the nondriver had a mean tau-dot
of $-0.58$ and performed comparably in all four conditions, suggesting that the tau-dot strategy has a general applicability.

It is interesting to note that the three women in the study had among the lowest mean tau-dot values ($-0.61$, $-0.60$, $-0.58$), suggesting that they were at the riskier end of the spectrum. This contrasts with previous reports of cautious behavior among women drivers (i.e., Hills, 1980). On the other hand, they also had the largest JNDS for tau-dot (0.33, 0.33), suggesting that they may be slightly less sensitive to optical expansion, consistent with earlier studies of time-to-contact (see McLeod & Ross, 1983; Schiff & Oldak, 1990). However, the present study was not designed to investigate gender differences in detail.

**Limitations**

An obvious limitation of the study is that the visual simulation excluded factors that might constrain strategies in actual driving. First, momentum was not a factor, so it was feasible to apply sharp decelerations that would be unlikely in actual driving, although this was limited by the brakes' maximal 0.7 g. Many of the slam-on-the-brakes and bang-bang trials followed this pattern. Second, in actual driving there are often vehicles approaching from behind, which would also make sudden deceleration unlikely. Third, the simulation was frictionless—if no deceleration was applied, the participant would continue to coast forever, lending itself to the bang-bang strategy. These factors could have encouraged impulsive strategies that were more effective in the simulation than in reality. Clearly, it would be desirable to replicate this study with an instrumented vehicle during actual driving.

Finally, the dynamics of the control device, and even the limb used to brake, could influence the behavior. In our displays, a constant force or brake position produced a constant deceleration, and it is possible that this favored strategies based on the optical consequences of holding deceleration constant, such as tau-dot drift. However, such control dynamics are not uncommon. The brake was modeled on the linear range of a typical car brake, and in general deceleration is linearly related to braking force for terrestrial locomotion. Further, the advantage of a tau-dot strategy is that it informs the observer about the adequacy of current deceleration, regardless of how that deceleration is produced. On the other hand, vehicles such as boats and helicopters may have quite different control dynamics, and braking strategies could be task-specific, particularly with limited control over deceleration. There are an indefinite number of possible control functions relating brake position and deceleration that could be investigated, and the present experiment is only a first step.

**Conclusion**

In sum, our results support the use of Lee's (1976) tau-dot strategy for the control of braking, even when spatial information is available. Observers appear to rely on a margin value $\tau_m = -0.5$ under a variety of conditions and to make brake adjustments that are, on average, equal to the difference between the current value and margin value of tau-dot, $\Delta \tau = (\tau_m - \tau)$. This differs from the negative results of Simpson (1991) and Glünder and Wagner (1992), possibly because our active control task provided a better assessment of performance. On the other hand, our findings are consistent with those of Kim et al. (1993), extending the perceptual boundary near $\tau = -0.5$ to the use of such a margin value in the active control of braking. These results imply that braking is not based on information about the 3-D structure of the scene, but is controlled on the basis of the optic flow variable $\tau$.

**References**


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Appendix

Braking Strategies: Derivations and Simulations

Simulations of the $\dot{\tau}$ Strategy

Our simulations of the tau-dot strategy (Figure 2) use initial conditions that are representative of those in the experiment: the middle values of initial distance (45 eye heights = 49.5 m) and initial time-to-contact (4.0 s), yielding an initial velocity of 12.4 m/s. The tau-dot equations for distance ($z$), velocity ($v$), and deceleration ($d$) are taken from Lee (1976):

$$ z = z_0(1 + \tau_m v d/z_0)^{-1/(1+\kappa_a)} $$

$$ v = v_0(1 + \tau_m v d/z_0)^{-1 + [1/(1+\kappa_a)]} $$

$$ d = (1 + \tau_m)(v_0^2/z_0)(1 + \tau_m v d/z_0)^{-2 + [2 + 1/(1+\kappa_a)]}, \quad (A1) $$

where $\tau_m$ is a "margin value" of tau-dot that the participant tries to maintain. The plots are computed from these equations for various values of $\tau_m$.

Derivation of the $\dot{\theta}$ Strategy

This strategy assumes that the driver simply holds the rate of expansion per se constant at some value $\dot{\theta} = k$, rather than holding $\tau$ constant. Assume that $0^\circ < \theta < 90^\circ$ and that the visual angle at time $t_0$ is $\theta_0$. Then the relation between $\theta$ and $k$ at any time $t$ is

$$ \theta = \theta_0 + k t. \quad (A2) $$

Additionally, from geometry we know:

$$ \theta = 2\tan^{-1}(r/z) $$

$$ z = r\tan(\theta/2). \quad (A3) $$

Substituting and differentiating, we see that at any time $t$, the $\dot{\theta}$ strategy would predict:

$$ z = r\tan[(\theta_0 + t)/2] $$

$$ v = -kr/2\sin[\dot{\theta}_0 + k t]/2] $$

$$ d = (k^2r/2)csc[(\theta_0 + k t)/2]\cos[(\theta_0 + k t)/2]. \quad (A4) $$

Additionally, we can compute the values of $\tau$ and $\dot{\tau}$ that would be predicted by the $\dot{\theta}$ strategy. Lee (1976) showed how $\tau$ and $\dot{\tau}$ can be calculated at any time from $z$, $v$, and $d$:

$$ \tau = z/v $$

$$ \dot{\tau} = -1 + (zd/v^2). \quad (A5) $$

Substituting the equations of motion for the $\dot{\theta}$ strategy into these equations, we find that many terms cancel. We are left with:

$$ \tau = (2k)\sin[(\theta_0 + k t)/2]\cos[(\theta_0 + k t)/2] $$

$$ \dot{\tau} = 1 + 2\cos^2[(\theta_0 + k t)/2]. \quad (A6) $$

Note that $\dot{\tau}$ should be close to $+1$ for small $\theta$, and decrease for larger $\theta$. For our displays, then, $\dot{\tau}$ should be close to $+1$ for most of the trial, decreasing somewhat as the object is approached. In fact, we can calculate that $\dot{\tau}$ should stay above 0.9 until the object occupies about 26° visual angle.

Simulations of the $\dot{\theta}$ Strategy

Our $\dot{\theta}$ simulations (Figure 3) use the same initial conditions as before; the middle values of initial distance (45 eye heights = 49.5 m) and time-to-contact (4.0 s), as well as the middle object size...


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(radius of center sign = 0.66 m). Initial velocity is again 12.4 m/s. According to this strategy, the observer would wait for a varying length of time until the expansion rate reached some value \( k \) before starting to brake, and then try to hold this value constant. The four curves in Figure 3 correspond to \( k \) values obtained by starting to brake at four different distances from the object: 0%, 25%, 50%, and 75% of the way from the object’s initial distance.

We can derive the corresponding values of \( k \) from the velocity equation in A4, using the subscript “b” to indicate the value at the onset of braking. Setting \( t = 0 \), we solve for \( k \):

\[
k = \left( -\frac{2v_b}{r} \right) \sin^2 \left( \frac{\theta_b}{2} \right),
\]

where

\[
\theta_b = 2\tan^{-1} \left( \frac{r}{z_b} \right).
\]

This yields values of \( k = 0.007, 0.012, 0.027, \) and 0.106, respectively.

**Derivation of Deceleration-Computing Strategies**

These strategies assume that the driver computes the necessary deceleration \( (d) \) at any given time from various combinations of the following variables (see Figure 1): \( z \) (distance to object), \( r \) (object size), \( \theta \) (current visual angle of object), \( \tau \) (the optical variable specifying time to contact). This approach assumes that these variables are visually available and are perceived accurately.

To reach a safe stop directly at an object, final velocity will by definition become zero exactly at the same time as the final position becomes zero \( (v_f = z_f = 0 \text{ at } t_f) \). Setting these conditions on the standard physical equations of motion, the first three computational strategies can be derived:

\[
\begin{align*}
v_f^2 &= v^2 - 2zd = 0 \\
v^2 &= 2zd \\
d &= \frac{v^2}{2z} \quad \text{(A8)}
\end{align*}
\]

\[
\begin{align*}
\frac{v_f}{t} = v - \tau d = 0; \quad \tau = t \\
v &= \tau d \\
d &= \frac{v}{\tau}. \quad \text{(A9)}
\end{align*}
\]

Substituting \( v = v/\tau \) yields:

\[
d = \frac{v}{\tau^2}. \quad \text{(A10)}
\]

Two additional strategies are possible if the size of the object is known. Substituting \( z = r\tan \left( \theta/2 \right) \) into Equation A10, we obtain

\[
d = r\left[ r^2 \tan \left( \theta/2 \right) \right]. \quad \text{(A11)}
\]

Substituting \( z = r\tan \left( \theta/2 \right) \) into Equation A8, we obtain

\[
d = \frac{(v^2/2r) \tan \left( \theta/2 \right)}{\tan \left( \theta/2 \right)}. \quad \text{(A12)}
\]

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**1996 APA Convention Call for Programs**

The Call for Programs for the 1996 APA annual convention appears in the September issue of the APA Monitor. The 1996 convention will be held in Toronto, Ontario, Canada, from August 9 through August 13. The deadline for receipt of program and presentation proposals is December 1, 1995. Additional copies of the Call are available from the APA Convention Office, effective in September. As a reminder, agreement to participate in the APA convention is now presumed to convey permission for the presentation to be audiotaped if selected for taping. Any speaker or participant who does not wish his or her presentation to be audiotaped must notify the person submitting the program either at the time the invitation is extended or before the December 1 deadline for proposal receipt.